

On the anomalous secular increase of the eccentricity of the orbit of the Moon

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ABSTRACT

A recent analysis of a Lunar Laser Ranging (LLR) data record spanning 38.7 yr revealed an anomalous increase of the eccentricity e of the lunar orbit amounting to $\dot{e}_{\text{meas}} = (9 \pm 3) \times 10^{-12} \text{ yr}^{-1}$. The present-day models of the dissipative phenomena occurring in the interiors of both the Earth and the Moon are not able to explain it. In this paper, we examine several dynamical effects, not modeled in the data analysis, in the framework of long-range modified models of gravity and of the standard Newtonian/Einsteinian paradigm. It turns out that none of them can accommodate \dot{e}_{meas} . Many of them do not even induce long-term changes in e ; other models do, instead, yield such an effect, but the resulting magnitudes are in disagreement with \dot{e}_{meas} . In particular, the general relativistic gravitomagnetic acceleration of the Moon due to the Earth's angular momentum has the right order of magnitude, but the resulting Lense-Thirring secular effect for the eccentricity vanishes. A potentially viable Newtonian candidate would be a trans-Plutonian massive object (Planet X/Nemesis/Tyche) since it, actually, would affect e with a non-vanishing long-term variation. On the other hand, the values for the physical and orbital parameters of such a hypothetical body required to obtain the right order of magnitude for \dot{e} are completely unrealistic. Moreover, they are in neat disagreement with both the most recent theoretical scenarios envisaging the existence of a distant, planetary-sized body and with the model-independent constraints on them dynamically inferred from planetary motions. Thus, the issue of finding a satisfactorily explanation for the anomalous behavior of the Moon's eccentricity remains open.

Subject headings: gravitation-Celestial mechanics-ephemerides-Moon-planets and satellites: general

1. Introduction

Anderson & Nieto (2010) recently reported an anomalous secular increase of the eccentricity¹ e of the orbit of the Moon

$$\dot{e}_{\text{meas}} = (9 \pm 3) \times 10^{-12} \text{ yr}^{-1} \quad (1)$$

based on an analysis of a long LLR data record spanning 38.7 yr (16 March 1970-22 November 2008) performed by Williams & Boggs (2009) with the suite of accurate dynamical force models of the DE421 ephemerides (Folkner et al. 2008; Williams et al. 2008) including all relevant Newtonian and Einsteinian effects. Notice that eq. (1) is statistically significant at a 3σ -level. Anderson & Nieto (2010) found that eq. (1) is not compatible with present, standard knowledge of dissipative processes in the interiors of both the Earth and Moon, which were, actually, modeled by Williams & Boggs (2009). The relevant physical and orbital parameters of the Earth and the Moon are reported in Table 1.

Table 1: Relevant physical and orbital parameters of the Earth-Moon system. a is the semimajor axis. e is the eccentricity. The inclination I refers to the mean ecliptic at J2000.0. Ω is the longitude of the ascending node and is referred to the mean equinox and ecliptic at J2000.0. ω is the argument of pericenter. G is the Newtonian gravitational constant. The masses of the Earth and the Moon are M and m , respectively. The orbital parameters of the Moon were retrieved from the WEB interface HORIZONS, by JPL, NASA, at the epoch J2000.0.

a (m)	e	I (deg)	Ω (deg)	ω (deg)	GM (m s ⁻²)	m/M
3.81219×10^8	0.0647	5.24	123.98	-51.86	3.98600×10^{14}	0.012

In this paper we look for a possible candidate for explaining such an anomaly in terms of both Newtonian and non-Newtonian gravitational dynamical effects, general relativistic or not.

To this aim, let us make the following, preliminary remarks. Naive, dimensional evaluations of the effect caused on e by an additional anomalous acceleration A can be made by noticing that

$$\dot{e} \approx \frac{A}{na}, \quad (2)$$

¹It is a dimensionless numerical parameter for which $0 \leq e < 1$ holds. It determines the shape of the Keplerian ellipse: $e = 0$ corresponds to a circle, while values close to unity yield highly elongated orbits.

with

$$na = 1.0 \times 10^3 \text{ m s}^{-1} = 3.2 \times 10^{10} \text{ m yr}^{-1} \quad (3)$$

for the geocentric orbit of the Moon. In it, a is the orbital semimajor axis, while $n \doteq \sqrt{\mu/a^3}$ is the Keplerian mean motion in which $\mu \doteq GM(1 + m/M)$ is the gravitational parameter of the Earth-Moon system: G is the Newtonian constant of gravitation. It turns out that an extra-acceleration as large as

$$A \approx 3 \times 10^{-16} \text{ m s}^{-2} = 0.3 \text{ m yr}^{-2} \quad (4)$$

would satisfy eq. (1). In fact, a mere order-of-magnitude analysis based on eq. (2) would be insufficient to draw meaningful conclusions: finding simply that this or that dynamical effect induces an extra-acceleration of the right order of magnitude may be highly misleading. Indeed, exact calculations of the secular variation of e caused by such putative promising candidate extra-accelerations A must be performed with standard perturbative techniques in order to check if they, actually, cause an averaged non-zero change of the eccentricity. Moreover, also in such potentially favorable cases caution is still in order. Indeed, it may well happen, in principle, that the resulting analytical expression for $\langle \dot{e} \rangle$ retains multiplicative factors² $1/e^k, k = 1, 2, 3, \dots$ or $e^k, k = 1, 2, 3, \dots$ which would notably alter the size of the found non-zero secular change of the eccentricity with respect to the expected values according to eq. (2).

The plan of the paper is as follows. In Section 2 we deal with several long-range models of modified gravity. Section 3 analyzes some dynamical effects in terms of the standard Newtonian/Einsteinian laws of gravitation. The conclusions are in Section 4.

2. Exotic models of modified gravity

2.1. A Rindler-type acceleration

As a practical example of the aforementioned caveat, let us consider the effective model for gravity of a central object of mass M at large scales recently constructed by Grumiller (2010). Among other things, it predicts the existence of a constant and uniform acceleration

$$\mathbf{A} = A_{\text{Rin}} \hat{\mathbf{r}} \quad (5)$$

radially directed towards M . As shown in Iorio (2010a), the Earth-Moon range residuals $\delta\rho$ over $\Delta t = 20 \text{ yr}$ yield the following constrain for a terrestrial Rindler-type extra-acceleration

$$A_{\text{Rin}} \lesssim 5 \times 10^{-16} \text{ m s}^{-2} = 0.5 \text{ m yr}^{-2}, \quad (6)$$

²Here e denotes the eccentricity: it is not the Napier number.

which is in good agreement with eq. (4).

The problem is that, actually, a radial and constant acceleration like that of eq. (5) does not induce any secular variation of the eccentricity. Indeed, from the standard Gauss³ perturbation equation for e (Bertotti et al. 2003)

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} \left\{ A_R \sin f + A_T \left[\cos f + \frac{1}{e} \left(1 - \frac{r}{a} \right) \right] \right\}, \quad (7)$$

in which f is the true anomaly⁴, and A_R, A_T are the radial and transverse components of the perturbing acceleration \mathbf{A} , it turns out (Iorio 2010a)

$$\Delta e = -\frac{A_{Rin} (1-e^2) (\cos E - \cos E_0)}{n^2}, \quad (8)$$

where E is the eccentric anomaly⁵, so that

$$\Delta e|_0^{2\pi} = 0. \quad (9)$$

2.2. A Pioneer-type acceleration

By mean of Pioneer-type acceleration we mean a constant and uniform extra-acceleration directed towards the Sun having a magnitude (Anderson et al. 1998, 2002; Anderson & Nieto 2010)

$$A_{\text{Pio}} = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2} = (8.70 \pm 1.32) \times 10^5 \text{ m yr}^{-2}. \quad (10)$$

In the present case, since we are in a geocentric ecliptic frame, such an acceleration becomes

$$\mathbf{A} = A_{\text{Pio}} \hat{\mathbf{n}}_{\odot}. \quad (11)$$

In it $\hat{\mathbf{n}}_{\odot} = \{\cos \lambda_{\odot}, \sin \lambda_{\odot}, 0\}$ is a unit vector pointing towards the Sun, where λ_{\odot} is its ecliptic longitude. It turns out that, actually, a non vanishing long-term variation of e occurs: it is

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{3A_{\text{Pio}}\sqrt{1-e^2}}{2na} [\sin \omega \cos(\Omega - \lambda_{\odot}) + \cos I \cos \omega \sin(\Omega - \lambda_{\odot})]. \quad (12)$$

³It is just the case to remind that the Gauss perturbative equations are valid for any kind of perturbing acceleration \mathbf{A} , whatever its physical origin may be.

⁴It is an angle counted from the pericenter, i.e. the point of closest approach to the central body, which instantaneously reckons the position of the test particle along its Keplerian ellipse.

⁵Basically, E can be regarded as a parametrization of the polar angle in the orbital plane.

Notice that eq. (12) contains the lunar longitude of the ascending node⁶ Ω , the argument of perigee⁷ ω and the inclination to the ecliptic I . It is well known (Roncoli 2005) that Ω and ω vary with periods of 18.6 yr and, 6.0 yr, respectively, with respect to the ecliptic, while I remains substantially constant. Thus, a slow modulation in $\langle \dot{e} \rangle$ would occur; in principle, it should have been detectable since the time span of the LLR data analysis amounts to $\Delta t = 38.7$ yr, but it is absent in eq. (1). Moreover, also the solar ecliptic longitude λ_{\odot} is present in eq. (12): it introduces a further, annual modulation which is absent in the empirical result of eq. (1). Actually, eq. (12) does not contain secular components.

It is just the case to remind that the existence of a Pioneer-type anomalous acceleration as large as in eq. (10) in the inner regions of the solar system is totally incompatible even with rather old planetary observations (Anderson et al. 2002). In the specific case of eq. (12), its amplitude would be of the order of $\langle \dot{e} \rangle \approx 10^{-5} \text{ yr}^{-1}$, in complete disagreement with eq. (1).

2.3. A Yukawa-type long-range modification of gravity

It is well known that a variety of theoretical paradigms (Adelberger et al. 2003; Bertolami & Páramos 2005) allow for Yukawa-like deviations from the usual Newtonian inverse-square law of gravitation (Burgess & Cloutier 1988). The Yukawa-type correction to the Newtonian gravitational potential $U_N = -\mu/r$, where $\mu \doteq GM$ is the gravitational parameter of the central body which acts as source of the supposedly modified gravitational field, is

$$U_Y = -\frac{\alpha\mu_{\infty}}{r} \exp\left(-\frac{r}{\lambda}\right), \quad (13)$$

where μ_{∞} is the gravitational parameter evaluated at distances r much larger than the scale length λ .

In order to compute the long-term effects of eq. (13) on the eccentricity of a test particle it is convenient to adopt the Lagrange perturbative scheme (Bertotti et al. 2003). In such a framework, the equation for the long-term variation of e is (Bertotti et al. 2003)

$$\left\langle \frac{de}{dt} \right\rangle = \frac{1}{na^2} \left(\frac{1-e^2}{e} \right) \left(\frac{1}{\sqrt{1-e^2}} \frac{\partial \mathcal{R}}{\partial \omega} - \frac{\partial \mathcal{R}}{\partial \mathcal{M}} \right), \quad (14)$$

⁶It is an angle in the reference $\{x, y\}$ plane determining the position of the line of the nodes, i.e. the intersection of the orbital plane with the reference $\{x, y\}$ plane, with respect to the reference x direction.

⁷It is an angle in the orbital plane reckoning the position of the point of closest approach with respect to the line of the nodes.

where ω is the argument of pericenter,

$$\mathcal{M} \doteq n(t - t_p) = E - e \sin E \quad (15)$$

is the mean anomaly of the test particle⁸, and \mathcal{R} denotes the average of the perturbing potential over one orbital revolution. In the case of a Yukawa-type perturbation, eq. (13) yields

$$\langle U_Y \rangle = -\frac{\alpha\mu_\infty \exp\left(-\frac{a}{\lambda}\right)}{a} I_0\left(\frac{ae}{\lambda}\right), \quad (16)$$

where $I_0(x)$ is the modified Bessel function of the first kind⁹ $I_k(x)$ for $k = 0$. An inspection of eq. (14) and eq. (16) immediately tells us that there is no secular variation of e caused by an anomalous Yukawa-type perturbation which, thus, cannot explain eq. (1).

2.4. Other long-range exotic models of gravity

The previous analysis has the merit of elucidating certain general features pertaining a vast category of long-range modified models of gravity. Indeed, eq. (14) tells us that a long-term change of e occurs only if the averaged extra-potential considered explicitly depends on ω and on time through M or, equivalently, E . Actually, the anomalous potentials arising in the majority of long-range modified models of gravity are time-independent and spherically symmetric (Dvali et al. 2000; Capozziello et al. 2001; Capozziello & Lambiase 2003; Dvali et al. 2003; Kerr et al. 2003; Allemandi et al. 2005; Gruzinov 2005; Jaekel & Reynaud 2005a,b; Navarro & van Acoleyen 2005; Reynaud & Jaekel 2005; Apostolopoulos & Tetradis 2006; Brownstein & Moffat 2006; Capozziello et al. 2006; Jaekel & Reynaud 2006a,b; Moffat 2006; Navarro & van Acoleyen 2006a,b; Sanders 2006; Adkins & McDonnell 2007; Adkins et al. 2007; Bertolami et al. 2007; Capozziello 2007; ?; Capozziello & Francaviglia 2008; Nojiri & Odintsov 2007; ?; Bertolami & Santos 2009; Hořava 2009a,b; Kehagias & Sfetsos 2009; de Felice & Tsujikawa 2010; Ruggiero 2010; Sotiriou & Faraoni 2010; Fabrina et al. 2011). Anomalous accelerations \mathbf{A} exhibiting a dependence on the test particle's velocity \mathbf{v} were also proposed in different frameworks (Jaekel & Reynaud 2005a,b; Kehagias & Sfetsos 2009). Since they have to be evaluated onto the unperturbed Keplerian ellipse, for which the following relations hold

⁸ t_p is the time of passage at pericenter.

⁹See on the WEB <http://mathworld.wolfram.com/ModifiedBesselFunctionoftheFirstKind.html> and references therein.

(Murray & Correia 2010)

$$\left\{ \begin{array}{l} r = a(1 - e \cos E), \\ dt = \left(\frac{1 - e \cos E}{n}\right) dE, \\ v_R = \frac{nae \sin E}{1 - e \cos E}, \\ v_T = \frac{na\sqrt{1 - e^2}}{1 - e \cos E}, \end{array} \right. \quad (17)$$

where v_R and v_T are the unperturbed, Keplerian radial and transverse components of \mathbf{v} , it was straightforward to infer from eq. (7) that no long-term variations of the eccentricity arose at all (Iorio 2007; Iorio & Ruggiero 2010).

An example of time-dependent anomalous potentials occurs if either a secular change of the Newtonian gravitational constant¹⁰ (Milne 1935; Dirac 1937) or of the mass of the central body is postulated, so that a percent time variation $\dot{\mu}/\mu$ of the gravitational parameter can be considered. In such a case, it was recently shown with the Gauss perturbative scheme that the eccentricity experiences a secular change given by (Iorio 2010b)

$$\langle \dot{e} \rangle = (1 + e) \left(\frac{\dot{\mu}}{\mu} \right). \quad (18)$$

As remarked in Iorio (2010b), eq. (1) and eq. (18) would imply an increase

$$\frac{\dot{\mu}}{\mu} = +8.5 \times 10^{-12} \text{ yr}^{-1}. \quad (19)$$

If attributed to a change in G , eq. (19) would be one order of magnitude larger than the present-day bounds on \dot{G}/G obtained from LLR (Müller & Biskupek 2007; Williams et al. 2007). Moreover, Pitjeva (2010) recently obtained a secular decrease of G as large as

$$\frac{\dot{G}}{G} = (-5.9 \pm 4.4) \times 10^{-14} \text{ yr}^{-1} \quad (20)$$

from planetary data analyses: if applied to eq. (18), it is clearly insufficient to explain the empirical result of eq. (1). Putting aside a variation of G , the gravitational parameter of the Earth may experience a time variation because of a steady mass accretion of non-annihilating Dark Matter (Blinnikov & Khlopov 1983; Khlopov et al. 1991; Khlopov 1999; Foot 2004; Adler 2008). Khriplovich & Shepelyansky (2009) and Xu & Siegel (2008) assume for the Earth

$$\frac{\dot{M}}{M} \approx +10^{-17} \text{ yr}^{-1}, \quad (21)$$

which is far smaller than eq. (19), as noticed by Iorio (2010c). Adler (2008) yields an even smaller figure for \dot{M}/M .

¹⁰According to Dirac (1937), G should decrease with the age of the Universe.

3. Standard Newtonian and Einsteinian dynamical effects

In this Section we look at possible dynamical causes for eq. (1) in terms of standard Newtonian and general relativistic gravitational effects which were not modeled in processing the LLR data.

3.1. The general relativistic Lense-Thirring field and other stationary spin-dependent effects

It is interesting to notice that the magnitude of the general relativistic Lense & Thirring (1918) acceleration experienced by the Moon because of the Earth’s angular momentum $S = 5.86 \times 10^{33} \text{ kg m}^2 \text{ s}^{-1}$ (McCarthy & Petit 2004) is just

$$A_{\text{LT}} \approx \frac{2vGS}{c^2a^3} = 1.6 \times 10^{-16} \text{ m s}^{-2} = 0.16 \text{ m yr}^{-2}, \quad (22)$$

i.e. close to eq. (4). On the other hand, it is well known that the Lense-Thirring effect does not cause long-term variations of the eccentricity. Indeed, (Soffel 1989)

$$\Delta e = -\frac{2GS \cos I (\cos f - \cos f_0)}{c^2na^3\sqrt{1-e^2}}. \quad (23)$$

In fact, eq. (23) holds only for a specific orientation of \mathbf{S} , which is assumed to be directed along the reference z axis. Actually, in Iorio (2010d) it was shown that e does not secularly change also for a generic orientation of \mathbf{S} since

$$\mathcal{R}_{\text{LT}} = \frac{2Gn}{c^2a(1-e^2)} [S_z \cos I + \sin I (S_x \sin \Omega - S_y \cos \Omega)]. \quad (24)$$

Thus, standard general relativistic gravitomagnetism cannot be the cause of eq. (1).

Iorio & Ruggiero (2009) explicitly worked out the gravitomagnetic orbital effects induced on the trajectory of a test particle by the the weak-field approximation of the Kerr-de Sitter metric. No long-term variations for e occur. Also the general relativistic spin-spin effects *à la* Stern-Gerlach do not affect the eccentricity with long-term variations (Iorio 2010d).

3.2. General relativistic gravitomagnetic time-varying effects

By using the Gauss perturbative equations, Ruggiero & Iorio (2010) analytically worked out the long-term variations of all the Keplerian orbital elements caused by general

relativistic gravitomagnetic time-varying effects. For the eccentricity, Ruggiero & Iorio (2010) found a non-vanishing secular change given by

$$\langle \dot{e} \rangle = -\frac{GS_1(2+e)\cos I}{c^2 a^3 n}, \quad (25)$$

in which S_1 denotes a linear change of the magnitude of the angular momentum of the central rotating body.

In the case of the Earth, Ruggiero & Iorio (2010) quote

$$S_1 = -5.6 \times 10^{16} \text{ kg m}^2 \text{ s}^{-2} \quad (26)$$

due to the secular decrease of the Earth’s diurnal rotation period (Brosche & Schuh 1998) $\dot{P}/P = -3 \times 10^{-10} \text{ yr}^{-1}$. Thus, eq. (25) and eq. (26) yield for the Moon’s eccentricity

$$\langle \dot{e} \rangle = -2 \times 10^{-23} \text{ yr}^{-1}, \quad (27)$$

which is totally negligible with respect to eq. (1).

3.3. The first and second post-Newtonian static components of the gravitational field

Also the first post-Newtonian, Schwarzschild-type, spherically symmetric static component of the gravitational field, which was, in fact, fully modeled by Williams & Boggs (2009), does not induce long-term variations of e (Soffel 1989). The same holds also for the spherically symmetric second post-Newtonian terms of order $\mathcal{O}(c^{-4})$ (Damour & Schäfer 1988; Schäfer & Wex 1993; Wex 1995), which were not modeled by Williams & Boggs (2009). Indeed, let us recall that the components of the spacetime metric tensor $g_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$, are, up to the second post-Newtonian order, (Nordtvedt 1996)

$$\begin{cases} g_{00} \cong 1 - 2\frac{\mathfrak{M}}{r} + 2\left(\frac{\mathfrak{M}}{r}\right)^2 - \frac{3}{2}\left(\frac{\mathfrak{M}}{r}\right)^3 + \dots, \\ g_{ij} \cong -\delta_{ij} \left[1 + 2\frac{\mathfrak{M}}{r} + \frac{3}{2}\left(\frac{\mathfrak{M}}{r}\right)^2 + \dots \right], \quad i, j = 1, 2, 3, \end{cases} \quad (28)$$

where $\mathfrak{M} \doteq \mu/c^2$. Notice that eq. (28) are written in the standard isotropic gauge, suitable for a direct comparison with the observations. Incidentally, let us remark that the second post-Newtonian acceleration for the Moon is just

$$A_{2\text{PN}} \approx \frac{\mu^2 n^2}{c^4 r} = 4 \times 10^{-25} \text{ m s}^{-2} = 4 \times 10^{-10} \text{ m yr}^{-2}. \quad (29)$$

3.4. The general relativistic effects for an oblate body

Soffel et al. (1988), by using the Gauss perturbative scheme and the usual Keplerian orbital elements, analytically worked out the first-order post-Newtonian orbital effects in the field of an oblate body with adimensional quadrupole mass moment J_2 and equatorial radius R .

It turns out that the eccentricity undergoes a non-vanishing long-term variation which, in general relativity, is (Soffel et al. 1988)

$$\langle \dot{e} \rangle = \frac{21nJ_2e \sin^2 I}{8(1-e^2)^3} \left(\frac{R}{a} \right)^2 \left(\frac{\mu}{c^2 a} \right) \left(1 + \frac{e^2}{2} \right) \sin 2\omega. \quad (30)$$

In view of the fact that, for the Earth, it is $J_2 = 1.08263 \times 10^{-3}$ (McCarthy & Petit 2004) and $R = 6.378 \times 10^3$ m (McCarthy & Petit 2004), it turns out that the first-order general relativistic $J_2 c^{-2}$ effect is not capable to explain eq. (1) since eq. (30) yields

$$\langle \dot{e} \rangle \leq 4 \times 10^{-19} \text{ yr}^{-1}. \quad (31)$$

Soffel et al. (1988) pointed out that the second-order mixed perturbations due to the Newtonian quadrupole field and the general relativistic Schwarzschild acceleration are of the same order of magnitude of the first-order ones: their orbital effects were analytically worked out by Heimberger et al. (1990) with the technique of the canonical Lie transformations applied to the Delaunay variables. Given their negligible magnitude, we do not further deal with them.

3.5. A massive ring of minor bodies

A Newtonian effect which was not modeled is the action of the Trans-Neptunian Objects of the Edgeworth-Kuiper belt (Edgeworth 1943; Kuiper 1951). It can be taken into account by means of a massive circular ring having mass $m_{\text{ring}} \leq 5.26 \times 10^{-8} M_{\odot}$ (Pitjeva 2010) and radius $R_{\text{ring}} = 43$ au (Pitjeva 2010). Following Fienga et al. (2008), it causes a perturbing radial acceleration

$$\mathbf{A}_{\text{ring}} = \frac{Gm_{\text{ring}}}{2rR_{\text{ring}}^2} \left[b_{\frac{3}{2}}^{(1)}(\alpha) - \alpha b_{\frac{3}{2}}^{(0)}(\alpha) \right] \mathbf{r}, \quad \alpha \doteq \frac{r}{R_{\text{ring}}}. \quad (32)$$

The Laplace coefficients are defined as (Murray & Dermott 1999)

$$b_s^{(j)} \doteq \frac{1}{\pi} \int_0^{2\pi} \frac{\cos(j\psi)}{(1 - 2\alpha \cos \psi + \alpha^2)^s} d\psi, \quad (33)$$

where s is a half-integer. Since for the Moon $\alpha \approx 3 \times 10^{-10}$, eq. (32) becomes

$$\mathbf{A}_{\text{ring}} \approx \frac{Gm_{\text{ring}}}{2rR_{\text{ring}}^2} \alpha \mathbf{r}, \quad (34)$$

with

$$A_{\text{ring}} \approx 10^{-23} \text{ m s}^{-2} \approx 10^{-8} \text{ m yr}^{-1}, \quad (35)$$

which is far smaller than eq. (4). Anyway, it can be shown by using the Gauss perturbative equation of eq. (7) that the long-term variation of e caused by eq. (32) vanishes.

The dynamical action of the belt of the minor asteroids (Krasinsky et al. 2002) was, actually, modeled, so that we do not consider it here.

3.6. A distant massive object: Planet X/Nemesis/Tyche

A promising candidate for explaining the anomalous increase of the lunar eccentricity may be, at least in principle, a trans-Plutonian massive body of planetary size located in the remote peripheries of the solar system: Planet X/Nemesis/Tyche (Lykawka & Mukai 2008; Melott & Bambach 2010; Fernández 2011; Matese & Whitmire 2011). Indeed, as we will see, the perturbation induced by it would actually cause a non-vanishing long-term variation of e . Moreover, since it depends on the spatial position of X in the sky and on its tidal parameter

$$\mathcal{K}_X \doteq \frac{Gm_X}{d_X^3}, \quad (36)$$

where m_X and d_X are the mass and the distance of X, respectively, it may happen that a suitable combination of them is able to reproduce the empirical result of eq. (1).

Let us recall that the perturbing potential due to a very distant, pointlike mass can be cast into the following quadrupolar form (Hogg et al. 1991)

$$U_X = \frac{\mathcal{K}_X}{2} \left[r^2 - 3 \left(\mathbf{r} \cdot \hat{\mathbf{l}} \right)^2 \right], \quad (37)$$

where $\hat{\mathbf{l}} = \{l_x, l_y, l_z\}$ is a unit vector directed towards X determining its position in the sky; its components are not independent since the constraint

$$l_x^2 + l_y^2 + l_z^2 = 1 \quad (38)$$

holds. By introducing the ecliptic latitude β_X and longitude λ_X , it is possible to write

$$\begin{cases} l_x = \cos \beta_X \cos \lambda_X, \\ l_y = \cos \beta_X \sin \lambda_X, \\ l_z = \sin \beta_X. \end{cases} \quad (39)$$

In eq. (37) $\mathbf{r} = \{x, y, z\}$ refers to the perturbed test particle. Iorio (2011) has recently shown that the average of eq. (37) over one orbital revolution is

$$\langle U_X \rangle = \frac{\mathcal{K}_X a^2}{32} \mathcal{U} \left(I, \Omega, \omega; \hat{\mathbf{l}} \right), \quad (40)$$

with

$$\begin{aligned} \mathcal{U} \doteq & - (2 + 3e^2) \left(-8 + 9l_x^2 + 9l_y^2 + 6l_z^2 \right) - 120e^2 \sin 2\omega \left(l_x \cos \Omega + l_y \sin \Omega \right) \left[l_z \sin I + \right. \\ & + \left. \cos I \left(l_y \cos \Omega - l_x \sin \Omega \right) \right] - 15e^2 \cos 2\omega \left[3 \left(l_x^2 - l_y^2 \right) \cos 2\Omega + 2 \left(l_x^2 + l_y^2 - 2l_z^2 \right) \sin^2 I - \right. \\ & - \left. 4l_z \sin 2I \left(l_y \cos \Omega - l_x \sin \Omega \right) + 6l_x l_y \sin 2\Omega \right] - 6 \left(2 + 3e^2 \right) \left[\left(l_x^2 - l_y^2 \right) \cos 2\Omega \sin^2 I + \right. \\ & + \left. 2l_z \sin 2I \left(l_y \cos \Omega - l_x \sin \Omega \right) + 2l_x l_y \sin^2 I \sin 2\Omega \right] - 3 \cos 2I \left\{ \left(2 + 3e^2 \right) \left(l_x^2 + l_y^2 - 2l_z^2 \right) + \right. \\ & + \left. 5e^2 \cos 2\omega \left[\left(l_x^2 - l_y^2 \right) \cos 2\Omega + 2l_x l_y \sin 2\Omega \right] \right\}. \end{aligned} \quad (41)$$

Note that eq. (40) and eq. (41) are exact: no approximations in e were used. In the integration $\hat{\mathbf{l}}$ was kept fixed over one orbital revolution of the perturbed test particle, as it is reasonable given the assumed large distance of X with respect to it.

The Lagrange planetary equation of eq. (14) straightforwardly yields (Iorio 2011)

$$\langle \dot{e} \rangle = \frac{15\mathcal{K}_X e \sqrt{1 - e^2}}{16n} \mathcal{E} \left(I, \Omega, \omega; \hat{\mathbf{l}} \right), \quad (42)$$

with

$$\begin{aligned} \mathcal{E} \doteq & -8l_z \cos 2\omega \sin I \left(l_x \cos \Omega + l_y \sin \Omega \right) + 4 \cos I \cos 2\omega \left[-2l_x l_y \cos 2\Omega + \right. \\ & + \left. \left(l_x^2 - l_y^2 \right) \sin 2\Omega \right] + \sin 2\omega \left[\left(l_x^2 - l_y^2 \right) \left(3 + \cos 2I \right) \cos 2\Omega + 2 \left(l_x^2 + l_y^2 - 2l_z^2 \right) \sin^2 I - \right. \\ & - \left. 4l_z \sin 2I \left(l_y \cos \Omega - l_x \sin \Omega \right) + 2l_x l_y \left(3 + \cos 2I \right) \sin 2\Omega \right]. \end{aligned} \quad (43)$$

Actually, the expectations concerning X are doomed to fade away. Indeed, apart from the modulation introduced by the presence of the time-varying ω and Ω in eq. (43), the values for the tidal parameter which would allow to obtain eq. (1) are too large for all the conceivable positions $\{\beta_X, \lambda_X\}$ of X in the sky. This can easily be checked by keeping ω and Ω fixed at their J2000.0 values as a first approximation.

Figure 1 depicts the X-induced variation of the lunar eccentricity, normalized to eq. (1), as a function of β_X and λ_X for the scenarios by Lykawka & Mukai (2008) ($m_X^{\max} = 0.7 m_\oplus$, $d_X^{\min} = 101.3$ au), and by Matese & Whitmire (2011) ($m_X^{\max} = 4 m_{\text{Jup}}$, $d_X = 30$ kau). It can be noticed that the physical and orbital features of X postulated by such two recent

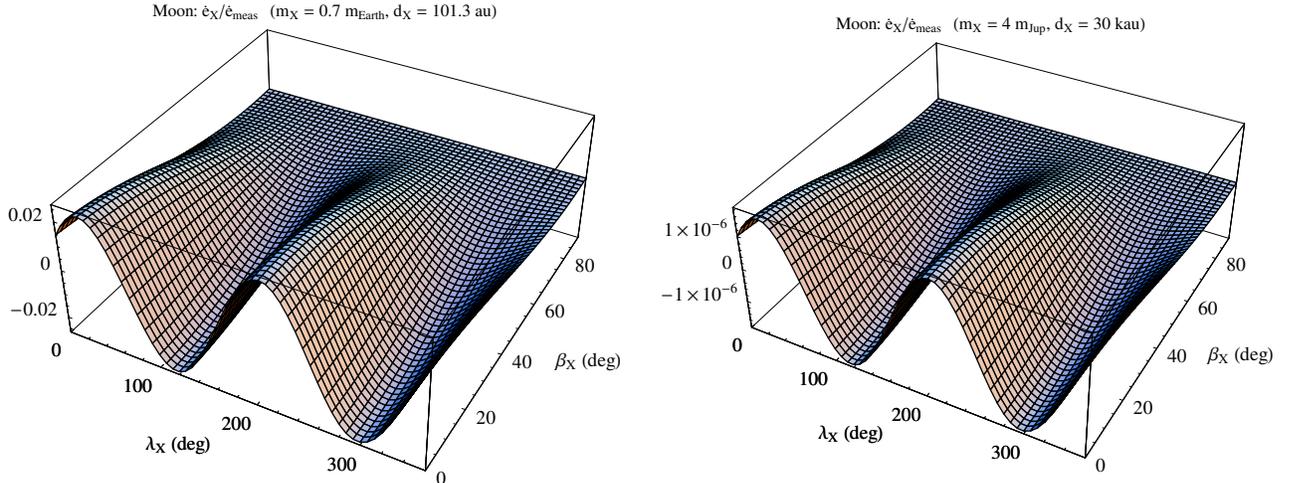


Fig. 1.— Long-term variation of the lunar eccentricity, normalized to eq. (1), induced by a trans-Plutonian, pointlike object X as a function of its ecliptic latitude β_X and longitude λ_X . The node Ω and the perigee ω of the Moon were kept fixed to the J2000.0 values quoted in Table 1. The scenarios for the perturbing body X are those by Lykawka & Mukai (2008) (left panel), and by Matese & Whitmire (2011) (right panel).

theoretical models would induce long-term variations of the lunar eccentricity much smaller than eq. (1). Conversely, it turns out that a tidal parameter as large as

$$\mathcal{K}_X = 4.46 \times 10^{-24} \text{ s}^{-2} \quad (44)$$

would yield the result of eq. (1). Actually, eq. (44) is totally unacceptable since it corresponds to distances of X as absurdly small as $d_X = 30$ au for a terrestrial body, and $d_X = 200$ au for a Jovian mass (Iorio 2011).

We must conclude that not even the hypothesis of Planet X is a viable one to explain the anomalous increase of the lunar eccentricity of eq. (1).

4. Summary and conclusions

In this paper we dealt with the anomalous increase of the eccentricity e of the orbit of the Moon recently reported from an analysis of a multidecadal record of LLR data points.

We looked for possible explanations in terms of unmodeled dynamical features of motion within either the standard Newtonian/Einsteinian paradigm or several long-range models of modified gravity. As a general rule, we, first, noticed that it would be misleading to simply find the right order of magnitude for the extra-acceleration due to this or that candidate effect. Indeed, it is mandatory to explicitly check if a potentially viable candidate

does actually induce a non-vanishing averaged variation of the eccentricity. This holds, in principle, for the search of an explanation of any other possible anomalous effect. Quite generally, it turned out that any time-independent and spherically symmetric perturbation does not affect the eccentricity with long-term changes.

Thus, most of the long-range modified models of gravity proposed in more or less recent times for other scopes are automatically ruled out. A Pioneer-like anomalous acceleration directed towards the Sun induces a long-term variation of e , but its magnitude is too large by several orders of magnitude. Conversely, the present-day limits on the magnitude of a terrestrial Rindler-type perturbing acceleration are of the right order of magnitude, but it does not secularly affect e . As time-dependent candidates capable to cause secular shifts of e , we considered the possible variation of the Earth’s gravitational parameter μ both because of a temporal variation of the Newtonian constant of gravitation G and of its mass itself due to a steady mass accretion of non-annihilating Dark Matter. In both cases, the resulting time variations of e are too small by several orders of magnitude.

Moving to standard general relativity, we found that the gravitomagnetic Lense-Thirring lunar acceleration due to the Earth’s angular momentum, not modeled in the data analysis, has the right order of magnitude, but it, actually, does not induce secular variations of e . The same holds also for other general relativistic spin-dependent effects. Conversely, e undergoes long-term changes caused by the general relativistic first-order effects due to the Earth’s oblateness, but they are far too small. The second-order post-Newtonian part of the gravitational field does not affect the eccentricity.

Within the Newtonian framework, we considered the action of a circular massive ring modeling the Edgeworth-Kuiper belt of Trans-Neptunian Objects, but it does not induce secular variations of e . In principle, a viable candidate would be a putative trans-Plutonian massive object (PlanetX/Nemesis/Tyche), recently revamped to accommodate certain features of the architecture of the Kuiper belt and of the distribution of the comets in the Oort cloud, since it would cause a non-vanishing long-term variation of the eccentricity. Actually, the values for its mass and distance needed to explain the empirically determined increase of the lunar eccentricity would be highly unrealistic and in contrast with the most recent viable theoretical scenarios for the existence of such a body. For example, a terrestrial-sized body should be located at just 30 au, while an object with the mass of Jupiter should be at 200 au.

Thus, in conclusion, the issue of finding a satisfactorily explanation of the observed orbital anomaly of the Moon still remains open. Our analysis should have effectively restricted the field of possible explanations, indirectly pointing towards either non-gravitational, mundane effects or some artifacts in the data processing. Further data analyses, hopefully performed by independent teams, should help in shedding further light on such an astrometric anomaly.

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